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Higher derivative four-fermion model in curved spacetime

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Abstract

We discuss the phase structure of a higher derivative four-fermion model in four dimensions in curved spacetime in frames of the $\frac{1}{N_c}$-expansion. First, we evaluate in our model the effective potential of two composite scalars in the linear curvature approximation using a local momentum representation in curved spacetime for the higher-derivative propagator which naturally appears. The symmetry breaking phenomenon and phase transition induced by curvature are numerically investigated. A numerical study of the dynamically generated fermionic mass, which depends on the coupling constants and on the cuvature, is also presented.

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1. Introduction. The celebrated idea by Nambu [1] to replace the fundamental Higgs boson in the standard model (SM) by a top quark condensate, which is responsible for dynamical symmetry breaking, has been investigated in some detail in refs. [2]. In frames of the Nambu-Jona-Lasinio mechanism [3], and under some conditions, it was shown that in the large-N limit the physics of such Nambu-Jona-Lasinio (NJL) model is equivalent to the physics of the SM with elementary scalar fields.

However, the original NJL model [3] with gauge fields and some of its modifications are considered now as non-renormalizable effective theories where the presence of an ultraviolet cut-off Λ at the loop diagrams is a necessary condition. It is clear that there are different possibilities to extend the NJL model. Among such possibilities, a quite interesting one is the introduction in the original Lagrangian of higher-derivative terms in the four fermion interaction [4, 5] or in the kinetic term [6]. It may be shown [4] that the physics of such generalized NJL models is still equivalent to the physics of the SM. Moreover, the additional advantage appears, that the ultraviolet cut-off Λ shows up at the Lagrangian level. That may help discuss the ambiguities which turn out in the choice of a cut-off [4, 5, 6] and violation of background gauge invariance. From another side, the inclusion of higher derivatives in effective theories gives the possibility to take into account the structural effects of the medium and external fields, like external gauge and gravitational fields or condensates.

It is the purpose of this work to consider a higher derivative four-fermion model in curved spacetime and to study the influence of an external gravitational field to the effective potential of composite scalars in the large-$\frac{1}{N_c}$ expansion. In particular, we discuss the phase structure and the dynamically generated fermion mass at non-zero curvature. This can be important in early Universe considerations. Notice that similar questions for the standard NJL model in curved spacetime have been already investigated in refs. [7, 8, 9] (for a study of $\lambda\phi^4$ theory see [11]). In particular, it was shown that curvature may induce first order phase transitions [8, 9]. We will here discuss whether these considerations go over to the higher derivative four fermion model at issue.

2. The model and its effective potential. We start by presenting the model which we set out to study (it was introduced in [5]). Its Lagrangian density in curved spacetime (for an introduction to quantum field theory in curved spacetime, see [12] is given by

$$\mathcal{L} = \bar{\psi} i\gamma^\mu(x)\nabla_\mu\psi + \frac{1}{4N_c\Lambda^2}\left\{\lambda_1\left(\bar{\psi}\psi\right)^2 + 3\lambda_2\left[\bar{\psi}\left(1-2\frac{\nabla^2}{\Lambda^2}\right)\psi\right]^2\right\} , \tag{1}$$

where N_c is the number of fermionic species, $\gamma^\mu(x)$ the Dirac matrices in curved space, Λ a cut-off parameter, and λ_1 and λ_2 are coupling constants. We work in the $\frac{1}{N_c}$-expansion scheme.

By introducing some auxiliary fields χ_1 and χ_2 we can give a description of this non-renormalizable theory by means of the action

$$S = \int d^4x\sqrt{g}\left\{\bar{\psi}i\gamma^\mu(x)\nabla_\mu\psi - N_c\Lambda^2\left(\frac{\chi_1^2}{\lambda_1}+\frac{\chi_2^2}{\lambda_2}\right) - \left[\chi_1\bar{\psi}\psi + \sqrt{3}\chi_2\bar{\psi}\left(1-2\frac{\not\nabla^2}{\Lambda^2}\right)\psi\right]\right\} \ . \quad (2)$$

Our purpose is to study the influence of external gravity on the dynamical breaking and restoration patterns of the symmetry possessed by the Lagrangian (1), which is given by the transformations $\psi \to \gamma_5\psi$ and $\bar{\psi} \to \bar{\psi}\gamma_5$. If we refer to the action (2), the symmetry is realized by adding the following transformations for the auxiliary fields: $\chi_1 \to -\chi_1$ and $\chi_2 \to -\chi_2$.

The effective action of this model in the $N_c \to \infty$ limit is given by

$$V_{eff} = \Lambda^2\left(\frac{\chi_2^2}{\lambda_1}+\frac{\chi_2^2}{\lambda_2}\right) + V_{1\ eff} \ , \quad (3)$$

with

$$V_{1\ eff} = i\,(\mathcal{V}ol)^{-1}\,\mathrm{Tr}\ \ln\left(i\not\nabla - m + 2\sqrt{3}\chi_2\frac{\not\nabla^2}{\Lambda^2}\right) \ ,$$

where $m = \chi_1+\sqrt{3}\chi_2$. Thus, to leading order in the $\frac{1}{N_c}$-expansion we have the gap equations, as follows

$$\left.\frac{\partial V_{1\ eff}}{\partial m}\right|_{\chi_2} = -i\,(\mathcal{V}ol)^{-1}\,\mathrm{Tr}\ \frac{1}{i\not\nabla - m + 2\sqrt{3}\chi_2\frac{\not\nabla^2}{\Lambda^2}} \ , \quad (4)$$

$$\left.\frac{\partial V_{1\ eff}}{\partial \chi_2}\right|_{m} = i\,(\mathcal{V}ol)^{-1}\,\mathrm{Tr}\ \frac{2\sqrt{3}\frac{\not\nabla^2}{\Lambda^2}}{i\not\nabla - m + 2\sqrt{3}\chi_2\frac{\not\nabla^2}{\Lambda^2}} \ . \quad (5)$$

First of all, we have to calculate the fermionic propagator, which satisfies

$$\left(i\not\nabla - m + 2\sqrt{3}\chi_2\frac{\not\nabla^2}{\Lambda^2}\right)\mathcal{G}(x,\,x') = \delta(x,\,x') \ .$$

Once this propagator is obtained (apart from terms which disappear under the Tr operation) it will be immediate to produce equations (4) and (5) in explicit form and also the dependence of the effective potential itself on the fields χ_1 and χ_2. The details of this rather lengthy derivation are written in the Appendix, where we give details about the calculation of the effective potential up to terms linear in the curvature.

The final outcome is shown below in natural variables, which are obtained by using the following definitions

$$r = \frac{\mathrm{R}}{\Lambda^2}\,, \quad x_1 = \frac{\chi_1}{\Lambda}\,, \quad x_2 = \frac{\chi_2}{\Lambda}\,, \quad a = x_1+\sqrt{3}x_2\,, \quad v = \frac{V_{eff}}{\Lambda^4} \ . \quad (6)$$

The 'dimensionless' effective potential v is given by

$$
\begin{aligned}
v \;=\;& \frac{a^2}{l_1} - \frac{a^2}{8\pi^2} + \frac{a^4}{32\pi^2} - \frac{a^2 r}{96\pi^2} - 2\sqrt{3}\frac{a x_2}{l_1} \\
&+\sqrt{3}\frac{a x_2}{4\pi^2} - \sqrt{3}\frac{a^3 x_2}{2\pi^2} + 3\frac{x_2^2}{l_1} + \frac{x_2^2}{l_2} - \frac{x_2^2}{2\pi^2} + 9\frac{a^2 x_2^2}{4\pi^2} + \\
&+\frac{r x_2^2}{16\pi^2} - 2\sqrt{3}\frac{a x_2^3}{\pi^2} + 9\frac{x_2^4}{4\pi^2} - \frac{a^4 \ln a^2}{16\pi^2} - \frac{a^2 r \ln a^2}{96\pi^2}\ .
\end{aligned} \tag{7}
$$

Note that in deriving this expression we have taken into account only terms up to quartic order on the fields a and x_2. The gap equations —which one obtains by differentiating v with respect to x_1 and x_2 and equating the results to zero— are, respectively

$$
\begin{aligned}
x_1\left(1-\frac{8\,\pi^2}{l_1}\right) \;=\;& -2\,3^{\frac{3}{2}}\,{x_1}^2\,x_2 - 18\,x_1\,{x_2}^2 - 8\sqrt{3}\,{x_2}^3 \\
&-a^3\,\ln a^2 - r\left(\frac{a}{6}+\frac{a\,\ln a^2}{12}\right)\ ,
\end{aligned} \tag{8}
$$

$$
\begin{aligned}
x_2\left(1-\frac{8\,\pi^2}{l_2}\right) \;=\;& -2\sqrt{3}\,{x_1}^3 - 18\,{x_1}^2\,x_2 - 8\,3^{\frac{3}{2}}\,x_1\,{x_2}^2 - 24\,{x_2}^3 \\
&-\sqrt{3}\,a^3\,\ln a^2 - r\left(\frac{x_1}{2\sqrt{3}}+\frac{a\,\ln a^2}{4\sqrt{3}}\right)\ .
\end{aligned} \tag{9}
$$

3. Symmetry breaking in curved spacetime. We can study now the influence of gravity on the symmetry breaking pattern of the theory. A simple inspection reveals that a positive curvature tends to protect the symmetry of the vacuum, and also a negative one may trigger the breaking of the symmetry.

In flat spacetime the symmetry is broken whenever either λ_1 or λ_2 are greater than $\frac{8\pi^2}{\Lambda^2}$, that is why we shall take in the sequel k_1 and k_2 to be $l_1 - 8\pi^2$ and $l_2 - 8\pi^2$ respectively. We may illustrate these remarks by studying the evolution of the minimum, given by two coordinates (which we choose to be a and x_2) in several circumstances. Before including gravity, it is worth discussing in more detail the situation in flat space. In fact, several cases may be analyzed (see [5]). To introduce this study it is convenient to define

$$
\pm\mu_i^2 \equiv \Lambda^2\left(1-\frac{8\pi^2}{\lambda_i^\pm}\right)\ ,
$$

where $\lambda_i^\pm$ means $\lambda_i > 8\pi^2$ (for the $+$ sign) or $\lambda_i < 8\pi^2$ (for the $-$ sign). One sticks here to the case when $\mu_i \ll \Lambda$. We only repeat the two situations given when $\mu_2^2 > 3\mu_1^2$ and $(\lambda_1^+,\ \lambda_2^-)$, or $\mu_2^2 < 3\mu_1^2$ and $(\lambda_1^-,\ \lambda_2^+)$. In both cases the results may be summarized by

$$
\chi_1^2 \;=\; \frac{\mu_1^2}{\ln\frac{\Lambda^2}{m^2}}\left|1-\frac{3\mu_1^2}{\mu_2^2}\right|^3\left[1+\mathcal{O}\left(\frac{1}{\ln\frac{\Lambda^2}{m^2}}\right)\right]\ , \tag{10}
$$

$$
\begin{aligned}
\chi_2 &= -\sqrt{3}\chi_1 \left(\frac{\mu_1}{\mu_2}\right)^2 \left[1+\mathcal{O}\left(\frac{1}{\ln\frac{\Lambda^2}{m^2}}\right)\right] , \\
m^2 &= \frac{\mu_1^2\mu_2^2}{\mid \mu_2^2 - 3\mu_1^2 \mid \, \ln\frac{\Lambda^2}{m^2}} .
\end{aligned}
$$

As a first example, consider the case in which the coupling constants are such that the symmetry is not broken in flat space, that is to say, $\lambda_i < 8\pi^2$. Let us see how the situation is modified as we move from negative to positive curvature. In all the figures, we have represented the values at the minimum of a and x_2, which correspond to the dynamically generated mass (m) and χ_2 transformed to natural units, as given in expression (6). We observe that there is a negative value of the curvature above which the symmetry is restored (see Figs. 1 and 2). We observe a continuous phase transition. Some comments about this fact are done below.

We can see the equivalent plots (Figs. 3 and 4) with the only difference that one of the couplings is such that the symmetry is broken at R = 0. One can identify a minimum positive value of the curvature for the symmetry to be restored. The character of the transition is still continuous.

This observation may be distressing if one recalls the results of [8, 9], where the NJL model was studied under the influence of a gravitational field in different situations. The conclusion of those works was always that if the coupling constant is greater than the critical value in flat space, there is a first order phase transition at some positive value of the curvature. In fact there is no actual contradiction. The point is that, till now, we have concentrated ourselves in cases where the coupling constants are around $8\pi^2$. We observe that if we explore regions in the space of parameters where λ_2 is much smaller, and λ_1 is greater than $8\pi^2$ —which would correspond to a limit where our theory approaches the Gross-Neveu model— then there is a first order phase transition for some positive value of the curvature.

Let us now illustrate what happens if we keep the curvature constant and vary the values of the couplings, to study how the domain which guarantees that the symmetry is not broken is modified by the presence of curvature. The situation of negative curvature is exemplified in Figs. 5 and 6. We see that the symmetry is broken from a negative value of k_1, in other words, the region of parameters of the theory where the symmetric vacuum is stable has shrinked.

To finish this discussion we consider the case of positive curvature. Here one can see that the aforementioned domain has been enlarged by the influence of positive curvature (see Figs. 7 and 8).

In summary, we have calculated the effective potential of composite scalars in the four-

fermion model (1) in curved spacetime, and numerically investigated the phase structure of the theory.

There is an interesting question about this model —namely whether it would be possible for some generalization of it to be represented in renormalizable form. As we see, if $\lambda_2 = 0$ in (1), this is perfectly feasible as long as one takes into account those terms in the field χ_1 which transform the theory into a renormalizable Yukawa-type model which describes near the critical point the physics of chiral symmetry breaking. As we see, the higher derivative term in [1] acts against such generalization at $\lambda_2 \neq 0$.

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Appendix: Expansion of the effective potential up to linear curvature terms

In this appendix we give details of the intermediate steps which are required in the lengthy calculation of the effective potential in the linear curvature approximation (for the representation of propagators in local momentum expansion and weak curvature, see [10]). The first step is the inversion of the operator given by

$$\left(i \not\nabla - m + 2\sqrt{3}\chi_2 \frac{\not\nabla^2}{\Lambda^2}\right)\left(i \not\nabla + m - 2\sqrt{3}\chi_2 \frac{\not\nabla^2}{\Lambda^2}\right) = -\left[\not\nabla^2 + \left(m - 2\sqrt{3}\chi_2 \frac{\not\nabla^2}{\Lambda^2}\right)^2\right] \tag{11}$$
$$= -\left[\left(\Box + \frac{\mathrm{R}}{4}\right)\left(1 - 4\sqrt{3}\chi_2 \frac{m}{\Lambda^2}\right) + m^2 + 12\frac{{\chi_2}^2}{\Lambda^4}\left(\Box + \frac{\mathrm{R}}{4}\right)^2\right] ,$$

where $\Box$ stands for the spinor d'Alembertian. Henceforth we shall use in our intermediate calculations, the relation

$$\mathrm{R}_{\alpha\beta\mu\nu} = \frac{\mathrm{R}}{n(n-1)}\left(g_{\alpha\mu}g_{\beta\nu} - g_{\alpha\nu}g_{\beta\mu}\right) , \tag{12}$$

which is valid in a constant-curvature spacetime. As we are restricting ourselves to the first nontrivial term in an expansion in the number of derivatives of the metric tensor, this apparent simplification will not introduce errors to this order. Again, keeping only terms proportional to the curvature, we may write

$$\Box = \partial^2 + \frac{\mathrm{R}}{3n(n-1)}\left(y^\mu y^\nu - y^2\eta^{\mu\nu}\right)\partial_\mu\partial_\nu + \frac{\mathrm{R}}{n(n-1)}\mathcal{J}^\mu{}_\lambda y^\lambda \partial_\mu + \frac{2}{3}\frac{\mathrm{R}}{n} y^\lambda \partial_\lambda , \tag{13}$$

being the y^μ Riemann normal coordinates, and

$$\left(i \not\nabla - m + 2\sqrt{3}\chi_2 \frac{\not\nabla^2}{\Lambda^2}\right)\left(i \not\nabla + m - 2\sqrt{3}\chi_2 \frac{\not\nabla^2}{\Lambda^2}\right) = \mathcal{A}_0 - \mathcal{A}_2 ,$$

where

$$\mathcal{A}_0 \equiv -\left[\partial^2\left(1-4\sqrt{3}\frac{\chi_2 m}{\Lambda^2}\right)+m^2+12\frac{\chi_2^2}{\Lambda^4}\partial^4\right] \ ,$$

and

$$\mathcal{A}_2 \equiv \mathrm{R}\left\{\left(1-4\sqrt{3}\frac{\chi_2 m}{\Lambda^2}\right)\left(\frac{1}{3n\,(n-1)}\left(y^\mu y^\nu - y^2\eta^{\mu\nu}\right)\partial_\mu\partial_\nu + \frac{\mathcal{J}^\mu_{\ \lambda}}{n\,(n-1)}y^\lambda\partial_\mu + \frac{2}{3\,n}y^\lambda\partial_\lambda + \frac{1}{4}\right) + 12\frac{\chi_2^2}{\Lambda^4}\left[\frac{\partial^2}{2} + \frac{2}{3}\frac{1}{n\,(n-1)}\left(y^\mu y^\nu - y^2\eta^{\mu\nu}\right)\partial_\mu\partial_\nu\partial^2 + \frac{2}{n\,(n-1)}\mathcal{J}^\mu_{\ \nu}y^\lambda\partial_\mu\partial^2 + \frac{2}{3n}\left(\partial^2 + 2y^\lambda\partial_\lambda\partial^2\right)\right]\right\} \ . \tag{14}$$

Now we write the equation satisfied by the inverse of the operator given in (12)

$$\left(\mathcal{A}_0 - \mathcal{A}_2 + \ldots\right)\mathrm{G}(y) = \delta(y)$$

The functional G may be given as an expansion in derivatives of the metric tensor $\mathrm{G} = \mathrm{G}_0 + \mathrm{G}_2 + \ldots$, where

$$\begin{aligned}\mathcal{A}_0\mathrm{G}_0(y) &= \delta(y), \\ \mathrm{G}_2 &= \mathrm{G}_0\mathcal{A}_2\mathrm{G}_0 \ .\end{aligned} \tag{15}$$

Let us give some explicit forms of these expressions:

$$\begin{aligned}\mathrm{G}_0 &= \int\frac{d^n\,k}{(2\pi)^n}\frac{\exp\left(-iky\right)}{k^2-\left(m+2\sqrt{3}\frac{\chi_2}{\Lambda^2}k^2\right)^2}, \\ \mathcal{A}_2\mathrm{G}_0 &= \int\frac{d^n\,k}{(2\pi)^n}\exp\left(-iky\right)\mathrm{R}\left\{\left(1-4\sqrt{3}\chi_2\frac{m}{\Lambda^2}\right)\left(\frac{1}{3\,n\,(n-1)}\left[n(n-1)g(k^2) + 2\,(2n+3)\,k^2g'(k^2) + 4\left(k^2\right)^2g''(k^2) - \left(4k^2f''(k^2)+2nf'(k^2)\right)\right]\right.\right. \\ &\quad \left. -\frac{2}{3n}\left(n\,g(k^2)+2k^2g(k^2)\right)+\frac{1}{4}\right) + 12\frac{\chi_2^2}{\Lambda^4}\left[\frac{1}{2}\bar{g}(k^2) + \frac{2}{3n(n-1)}\left[n(n+1)\bar{g}(k^2) + 2k^2(2n+3)\bar{g}'(k^2) + 4k^2\bar{g}'' - \left(4k^2\bar{f}''(k^2) + 2n\bar{f}'(k^2)\right)\right]\right. \\ &\quad \left.\left. + \frac{2}{3n}\left(\bar{g}(k^2) - 2n\left(\bar{g}(k^2)+2k^2\bar{g}(k^2)\right)\right)\right]\right\} \ .\end{aligned} \tag{16}$$

Some definitions are in order:

$$\begin{aligned}g(x) &\equiv \frac{1}{x-\left(a+2\sqrt{3}\chi_2\frac{x}{\Lambda^2}\right)^2} \ , \\ f(x) &\equiv \frac{x}{x-\left(a+2\sqrt{3}\chi_2\frac{x}{\Lambda^2}\right)^2} \ , \\ \bar{f}(x) &\equiv \frac{-x^2}{x-\left(a+2\sqrt{3}\chi_2\frac{x}{\Lambda^2}\right)^2} \ , \\ \bar{g}(x) &\equiv \frac{-x}{x-\left(a+2\sqrt{3}\chi_2\frac{x}{\Lambda^2}\right)^2} \ .\end{aligned} \tag{17}$$

Now $\mathcal{G} = \mathcal{B}\mathrm{G}$, where $\mathcal{B} \equiv i \not\nabla + a - 2\sqrt{3}\chi_2 \frac{\nabla^2}{\Lambda^2}$, and we may expand $\mathcal{B} = \mathcal{B}_0 + \mathcal{B}_2 + \ldots$ (again according to the number of derivatives of the metric tensor). The explicit expression of the operators $\mathcal{B}_0$ and $\mathcal{B}_2$ are:

$$\begin{aligned}
\mathcal{B}_0 &= i \hat{\not\partial} + a - 2\sqrt{3}\chi_2 \frac{\partial^2}{\Lambda^2} \ , \qquad (18)\\
\mathcal{B}_2 &= \frac{i\mathrm{R}}{6n(n-1)} \left(y^2 \ \hat{\not\partial} - \not y y^\lambda \partial_\lambda \right) + \frac{i\mathrm{R}}{2n(n-1)} y_a \hat{\gamma}_b \mathcal{J}^{ab} \\
&\quad -2\sqrt{3}\chi_2 \frac{\mathrm{R}}{\Lambda^2} \left(\frac{1}{3n(n-1)} \left(y^\mu y^\nu - y^2 \eta^{\mu\nu} \right) \partial_\mu \partial_\nu + \frac{\mathcal{J}^\mu_\nu}{n(n-1)} y^\lambda \partial_\mu \right. \\
&\quad \left. + \frac{2}{3n} y^\lambda \partial_\lambda + \frac{1}{4} \right) \ ,
\end{aligned}$$

where the symbol $\hat{}$ means that the Dirac matrices involved satisfy the anticommutation relations $\{\hat{\gamma}^a, \ \hat{\gamma}^b\} = \eta^{ab}$.

We need only to expand the expressions shown below

$$\mathcal{G}_0 = \mathcal{B}_0 \mathrm{G}_0 \ , \qquad (19)$$

$$\mathcal{G}_2 = \mathcal{B}_2 \mathrm{G}_0 + \mathcal{B}_0 \mathrm{G}_0 \mathcal{A}_2 \mathrm{G}_0 \ . \qquad (20)$$

In particular, the explicit form of $\mathcal{G}_0$ and $\mathcal{B}_2\mathrm{G}_0$ are

$$\mathcal{G}_0 = \int \frac{d^n k}{(2\pi)^n} \frac{\exp\left(-iky\right)}{\hat{\not k} - \left(a + 2\sqrt{3}\frac{\chi_2}{\Lambda^2} k^2 \right)^2} \ , \qquad (21)$$

$$\mathcal{B}_2 \mathrm{G}_0 = -\int \frac{d^n k}{(2\pi)^n} \exp\left(-iky\right) 2\sqrt{3}\chi_2 \frac{\mathrm{R}}{\Lambda^2} \left(\frac{1}{3n(n-1)} \mathrm{h1}(k^2) - \frac{2}{3n} \mathrm{h2}(k^2) + \frac{1}{4} \right) \ . \qquad (22)$$

Equation (22) is true, apart from terms which have an odd number of $\hat{\gamma}$ matrices —and which will not contribute to the effective potential. Finally, h1 and h2 are defined as:

$$\begin{aligned}
\mathrm{h1}(k^2) &= n(n+1) g(k^2) + 2k^2 (2n+3) g'(k^2) + \qquad (23)\\
&\quad + 4 \left(k^2 \right)^2 g''(k^2) - \left(4k^2 f''(k^2) + 2n f'(k^2) \right) \ , \\
\mathrm{h2}(k^2) &= n g(k^2) + 2k^2 g'(k^2) \ .
\end{aligned}$$

Putting all these pieces together we may construct the propagator $\mathcal{G}$ (apart from terms with an odd number of $\hat{\gamma}$ matrices) and perform the momentum integrations ensuing from the Tr operations in (4) and (5). As for (5), remember that $\nabla^2 = \Box + \frac{\mathrm{R}}{4}$ and $\Box$ is given in (13).

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Figure captions

Fig. 1 The coupling constants are kept constant at the values given by $k_1 = -2 = k_2$. One sees a continuous phase transition.

Fig. 2 The coupling constants are kept constant at the values given by $k_1 = -2 = k_2$. There is a continuous phase transition at some negative value of the curvature.

Fig. 3 The coupling constants are kept constant at the values given by $k_1 = -1,\ k_2 = 0.2$. Now we see a continuous phase transition.

Fig. 4 The coupling constants are kept constant at the values given by $k_1 = -1\ k_2 = 0.2$.

Fig. 5 The curvature is such that $r = -0.0008$ and k_2 is kept constant at $k_2 = -0.8$.

Fig. 6 The curvature is such that $r = -0.0008$ and k_2 is kept constant at $k_2 = -0.8$.

Fig. 7 The curvature is such that $r = 0.0008$ and k_2 is kept constant at $k_2 = -0.8$. Now the range of values of the coupling constants which maintain the symmetry of the vacuum is enlarged as the curvature gets positive.

Fig. 8 The curvature is such that $r = 0.0008$ and k_2 is kept constant at $k_2 = -0.8$.

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http://arXiv.org/ps/hep-th/9411216v1

Fig. 2

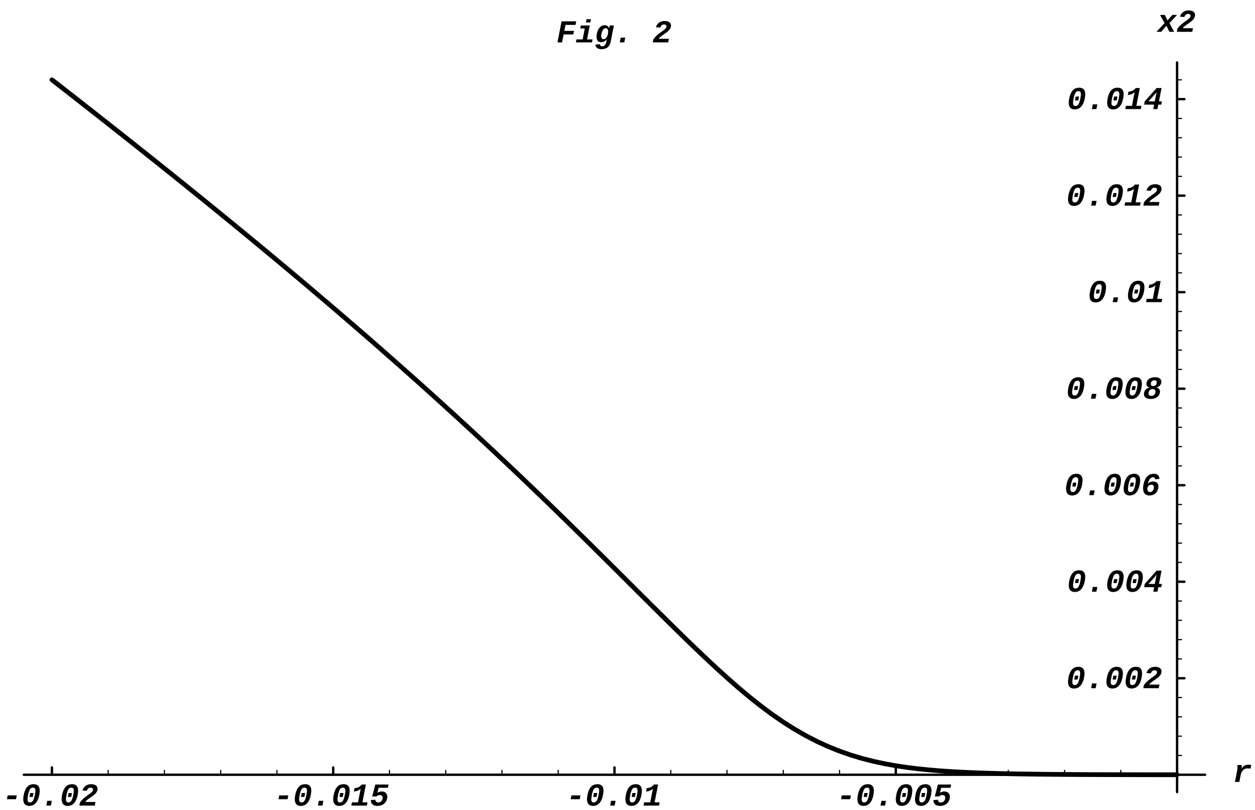


Fig. 3

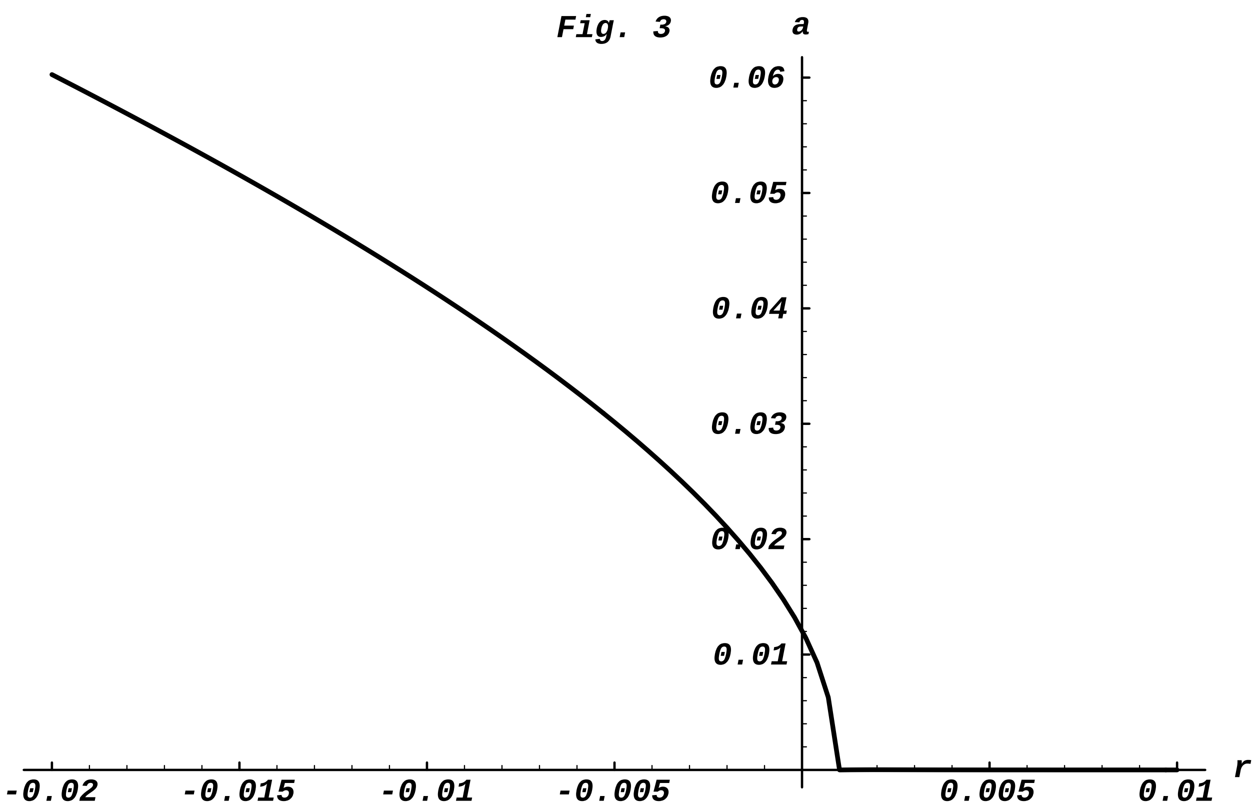


Fig. 4

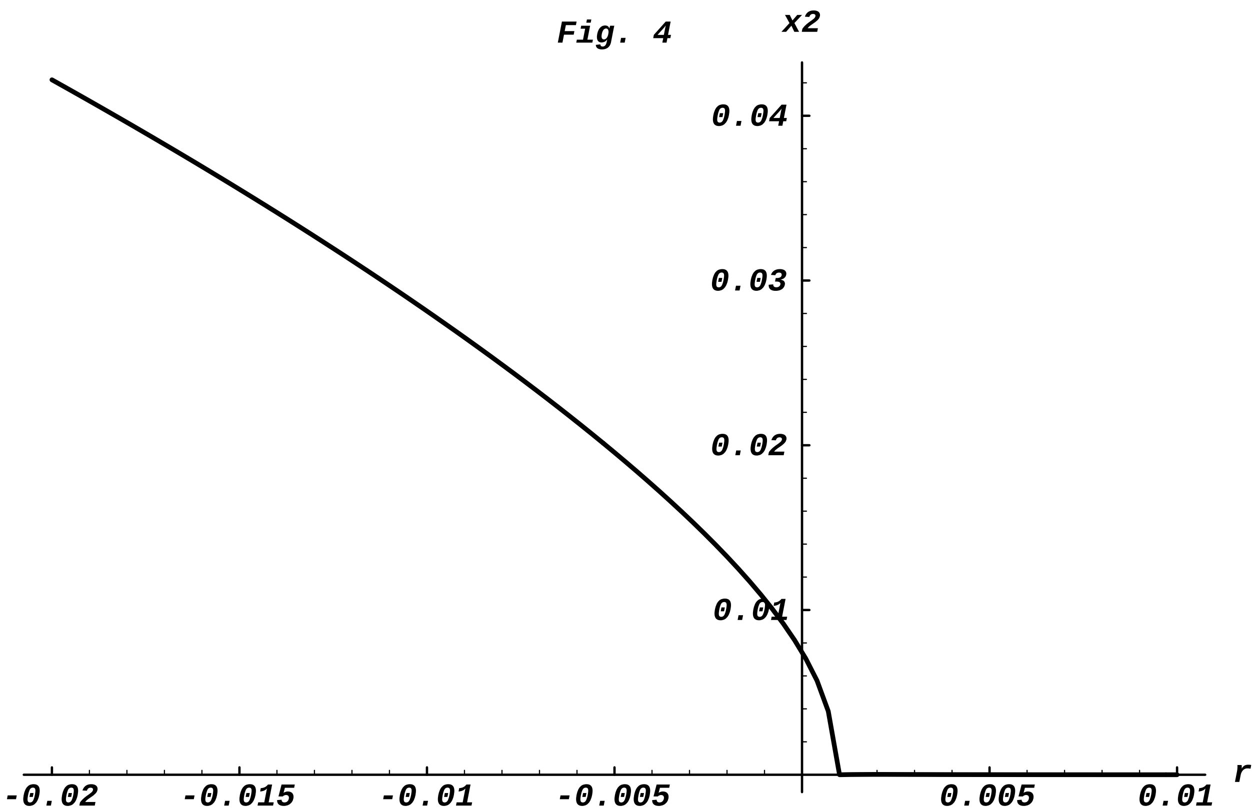


Fig. 5

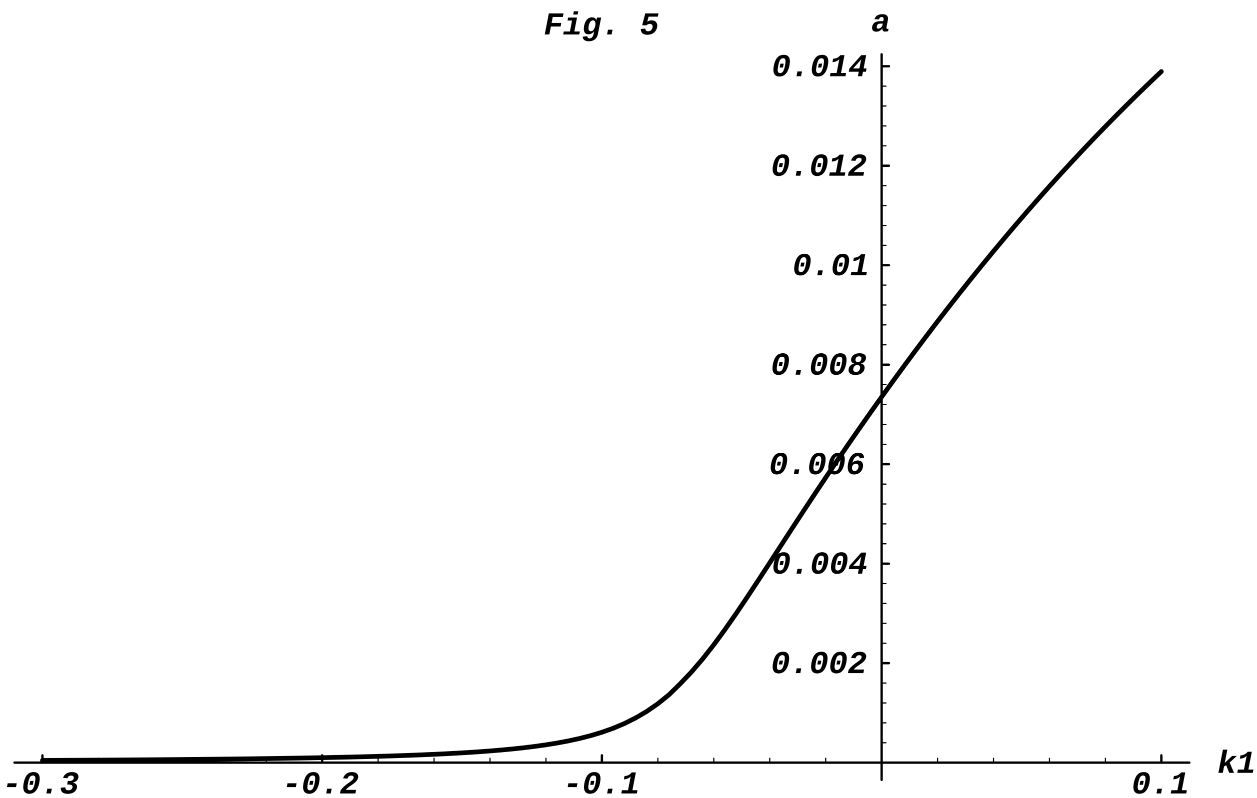


Fig. 6

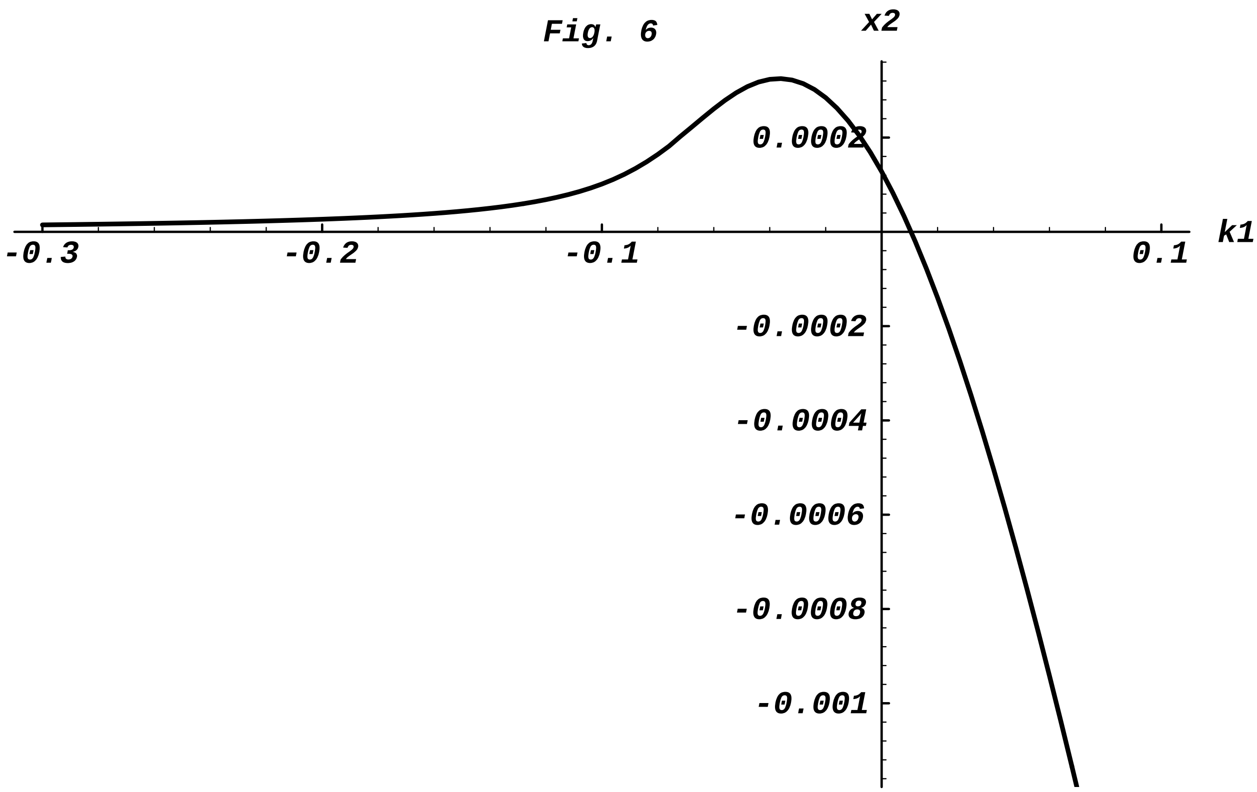
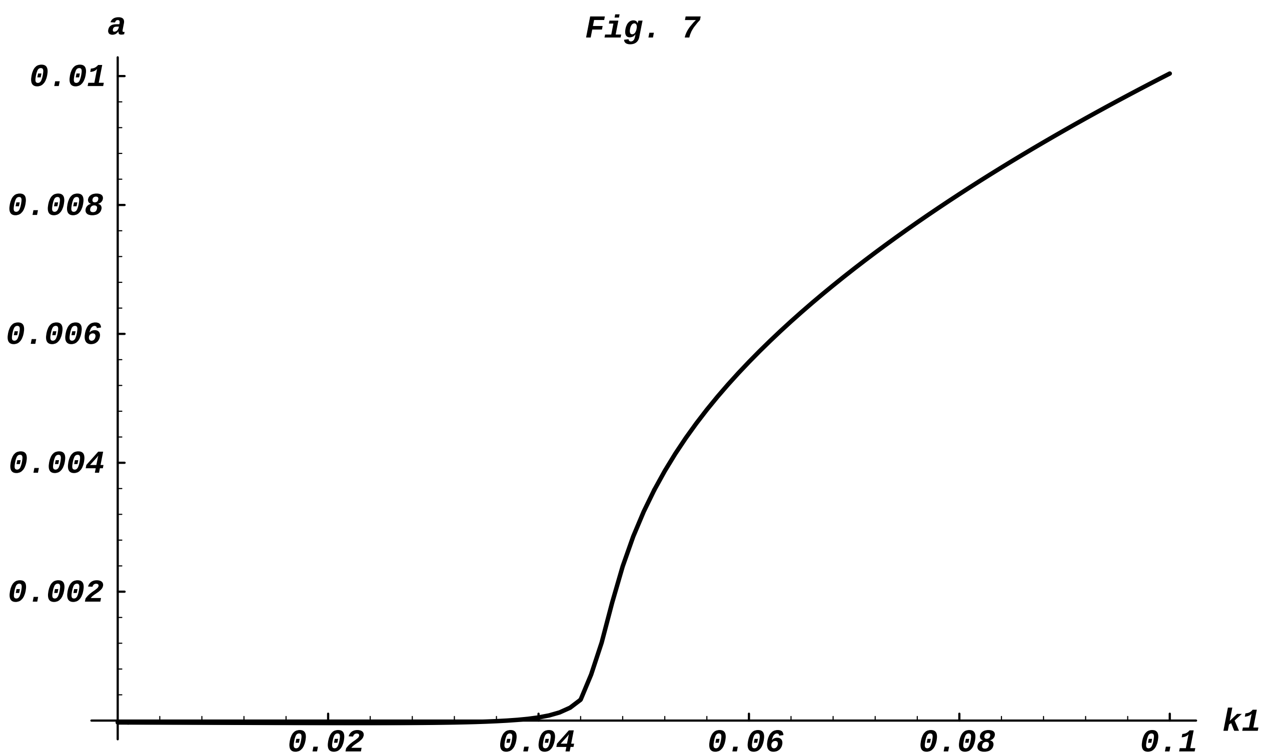


Fig. 7



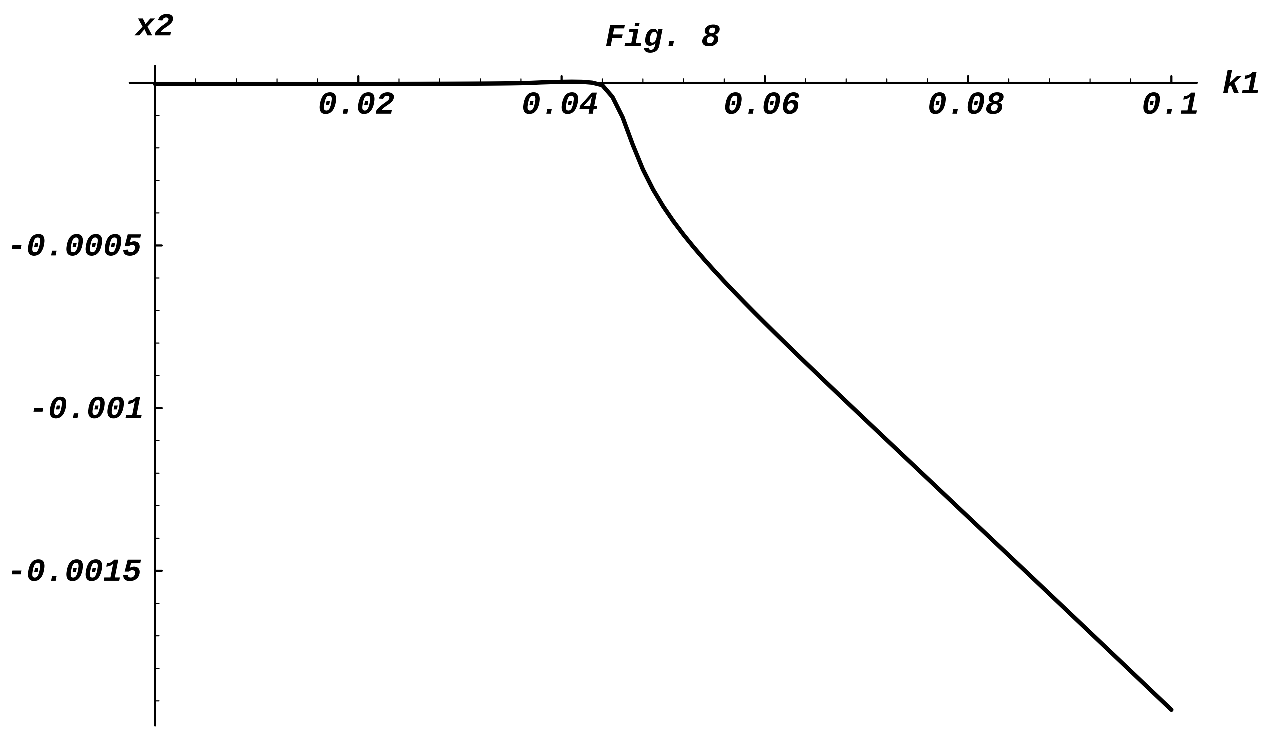

x2
Fig. 8
k1
0.02
0.04
0.06
0.08
0.1
-0.0005
-0.001
-0.0015

Fig. 1